MASTER IN ACTUARIAL SCIENCE

Risk Models

20/01/2017

Time allowed: 3 hours

Instructions:

1. This paper contains 8 questions and comprises 3 pages including the title page.
2. Enter all requested details on the cover sheet.
3. You have 10 minutes reading time. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all 8 questions.
6. Begin your answer to each of the 8 questions on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. The distributed formulary and the Formulae and Tables for Actuarial Examinations (the 2002 edition) may be used. Note that the parametrization used for the different distributions is that of the distributed formulary.

1. You observed a random sample of 10 claims from a portfolio where there is no deductible and the policy limit varies by policy. The sample is (350 350 500 500 500^+ 1000 1000^+ 1000^+ 1200 1500) where the symbol + indicates that the loss exceeds the policy limit.
    a. **[15]** Obtain $\hat{S}_1(1250)$, the product-limit estimate of $S(1250)$ and also compute a 90% linear confidence interval for $S(1250)$. Comment.
    b. **[10]** Explain the meaning of ${}_{250}p_{1000}$ and estimate it.
    c. **[5]** Estimate $S(1250)$ using Nelson-Aalen approach.
    d. **[15]** Let $\hat{S}_2(1250)$ be the maximum likelihood estimate of $S(1250)$ under the assumption that the losses follow an exponential distribution. Compute it.

2. **[15]** You know that a $(1-\alpha)$ log-transformed confidence interval for $H(t)$ is given by $(0.7,0.9)$. Obtain a linear confidence interval for $H(t)$ with the same confidence level.

3. **[15]** You observed the sample (2 2 3 3 3 3 3 3 5 5) from a population with density function $f(x)$. Using a gamma kernel with $\alpha=2$ obtain an estimate for $f(4)$ and for $S(4)$. Note: you can use the fact that, when α is integer, the distribution function of a gamma with parameters $\alpha=2$ and θ can be written as $F(x|\alpha=2,\theta)=1-\left(1+\dfrac{x}{\theta}\right)e^{-x/\theta}$.

4. **[15]** A random sample of claims has been drawn from an Inverse Pareto distribution. The 11 observed values are (1.1, 2.1, 4.0, 8.0, 9.8, 11.8, 13.3, 23.1, 36.2, 150.9, 213.2). Our purpose is to estimate the parameters τ and θ using the percentiles adjustment method (percentiles 30% and 70%). Write the equations that need to be solved to obtain these estimates and explain how to solve the system using EXCEL. You can take advantage of the page where an EXCEL sheet and solver input screen are printed

5. The random variable X has density function given by $f(x|\theta)=\theta^{-2}\,x\,e^{-0.5x^2/\theta^2}$, $x>0$, $\theta>0$. You also know that $E(X)=(\theta/2)\sqrt{2\pi}$. You are given a random sample with 5 observations (5; 2; 3; 7; 4)
    a. **[15]** Obtain the maximum likelihood estimate for θ and also determine the moment estimate for θ.
    b. **[10]** Obtain a maximum likelihood estimate for the 95% *VaR* of X and also obtain an approximate estimate for the variance of this estimator.

c. **[15]** Now you are warned that observations 1 and 5 are censored observations, i.e the corresponding value of the random variable is at least 5 and 4 respectively. You also are informed that observation 2 is truncated at 1 and observation 3 is truncated at 2. Obtain a new estimate for θ.

6. Assume that we observed a random sample $(x_1, x_2, \cdots, x_n)$, with $n = 50$ and $\bar{x} = 1.3$, from a population with density function given by $f(x \mid \theta) = \dfrac{\theta^4}{6} x^3 e^{-\theta x}$, $x > 0$, $\theta > 0$. Using a Bayesian framework, we define our prior as $\pi(\theta) = \dfrac{1}{25} \theta e^{-\theta/5}$, $\theta > 0$.
   a. **[10]** Obtain the posterior distribution for θ. Note: if you are unable to compute the posterior assume – and clearly state it – that the posterior distribution is a gamma distribution with parameters 57 and 0.8 (which is not the correct answer) to answer to the remaining questions.
   b. **[10]** Compute a Bayes estimate for θ assuming a quadratic loss function and compute another estimate assuming a 0-1 loss function.
   c. **[10]** Using Bayes Central Limit Theorem, obtain an approximate HPD interval (95%).

7. **[15]** A sample of claim amounts is (42 106 357 895 1334). Test (5% significance level) if it is acceptable to consider that the claim amounts were generated by an inverse Pareto distribution with parameters $\tau = 3$ and $\theta = 100$.

8. Assume that the number of claims by year of a given policy, is Poisson distributed with parameter 0.1 and that the claim amounts , $\{X_i\}$, $i = 1, 2, \cdots, N$, can be assumed to be independent Pareto distributed random variables with parameters $\alpha = 3$ and $\theta = 200$ and are also independent of N.
   a. **[15]** Explain how to use simulation to approximate the distribution of the aggregate claim amount of the policy, $Y = \sum_{i=0}^{N} X_i$, with $X_0 \equiv 0$. Also explain how to estimate $P(Y > 500)$.
   b. **[10]** Assuming that the first 5 pseudo-random generated values are $0.95 \quad 0.85 \quad 0.53 \quad 0.10 \quad 0.05$ what is the generated value for the aggregate claim amount of the first replica?

SOLUTION

1.

a) KM approach

y	350	500	1000	1200	1500
s	2	2	1	1	1
r	10	8	5	2	1

$$\hat{S}_1(1250) = \sum_{j:y_j \le 1250} \frac{r_j - s_j}{r_j} = \frac{8}{10} \times \frac{6}{8} \times \frac{4}{5} \times \frac{1}{2} = 0.24$$

Greenwood's approximation:

$$\hat{\text{var}}(\hat{S}_1(1250)) \approx \hat{S}_1(1250)^2 \sum_{j:y_j \le 1250} \frac{s_j}{r_j\,(r_j - s_j)} = 0.24^2 \times 0.6167 = 0.03552$$

Then the 90% linear CI is $0.24 \pm 1.645 \times \sqrt{0.03552}$, i.e. (-0.07; 0.55). As $S(1250)$ is a probability and then bounded by 0 and 1, the interval needs to be corrected (0; 0.55).

b) The meaning of ${}_{250}p_{1000}$ is the probability that a claim is greater than 1250 given that it is greater than 1000.

$${}_{250}p_{1000} = P(X > 1250 \mid X > 1000) = \frac{S(1250)}{S(1000)}$$

$${}_{250}\hat{p}_{1000} = \frac{\hat{S}_1(1250)}{\hat{S}_1(1000)} = \frac{0.24}{0.48} = 0.5$$ as $$\hat{S}_1(1000) = \sum_{j:y_j \le 1000} \frac{r_j - s_j}{r_j} = \frac{8}{10} \times \frac{6}{8} \times \frac{4}{5} = 0.48$$

c) Nelson-Aalen approach

$$\hat{H}(1250) = \sum_{j:y_j \le 1250} \frac{s_j}{r_j} = \frac{2}{10} + \frac{2}{8} + \frac{1}{5} + \frac{1}{2} = 1.15$$

Then $\hat{S}(1250) = e^{-\hat{H}(1250)} = 0.3166$

d) ML approach

$f(x \mid \theta) = \theta^{-1} e^{-x/\theta}$, $x > 0$, $\theta > 0$

$$\ell(\theta) = \sum_{i=1}^{7} \ln f(x_{(i)} \mid \theta) + \ln S(500 \mid \theta) + 2 \times \ln S(1000 \mid \theta)$$

$$= -7\ln\theta - \frac{1}{\theta}\sum_{i=1}^{7} x_{(i)} - \frac{500}{\theta} - 2 \times \frac{1000}{\theta} = -7\ln\theta - \frac{5400}{\theta} - \frac{500}{\theta} - \frac{2000}{\theta}$$

$$= -7\ln\theta - \frac{7900}{\theta}$$

$\ell'(\theta) = -\dfrac{7}{\theta} + \dfrac{7900}{\theta^2}$ and $\ell'(\theta) = 0 \Leftrightarrow \dfrac{7}{\theta} = \dfrac{7900}{\theta^2} \Leftrightarrow \theta = \dfrac{7900}{7}$

$\ell''(\theta) = \dfrac{1}{\theta^2}\left(7 - \dfrac{15800}{\theta}\right)$ and then $\ell''\left(\dfrac{7900}{7}\right) = \dfrac{1}{\theta^2}\left(7 - \dfrac{15800 \times 7}{7900}\right) < 0$

The ML estimate of $S(1250)$ is then $\hat{S}_2(1250) = e^{-1250/\hat{\theta}} = e^{-1250 \times 7/7900} = 0.3304$

2.

Let (a,b) be the log-transformed interval for $H(t)$. Then $\hat{H}(t)/U = a$ and $\hat{H}(t)\times U = b$ where $U = \exp\left(z_{\alpha/2} \frac{\sqrt{\hat{\mathrm{var}}(\hat{H}(t))}}{\hat{H}(t)} \right)$

From the first 2 equations we get $\hat{H}(t) = \sqrt{ab}$ and $U = \frac{\hat{H}(t)}{a} = \sqrt{\frac{b}{a}}$. Then

$$\sqrt{\frac{b}{a}} = \exp\left(z_{\alpha/2} \frac{\sqrt{\hat{\mathrm{var}}(\hat{H}(t))}}{\hat{H}(t)} \right) \Leftrightarrow \frac{1}{2}\ln\left(\frac{b}{a}\right) = z_{\alpha/2}\frac{\sqrt{\hat{\mathrm{var}}(\hat{H}(t))}}{\hat{H}(t)} \Leftrightarrow \sqrt{\hat{\mathrm{var}}(\hat{H}(t))} = \frac{\sqrt{ab}}{2z_{\alpha/2}}\ln\left(\frac{b}{a}\right)$$

Consequently the linear confidence interval for $\hat{H}(t)$, given by $\hat{H}(t) \pm z_{\alpha/2}\sqrt{\hat{\mathrm{var}}(\hat{H}(t))}$, will be $\hat{H}(t) \pm z_{\alpha/2}\frac{\sqrt{ab}}{2z_{\alpha/2}}\ln(a/b)$, i.e. $\sqrt{ab} \pm \frac{\sqrt{ab}}{2}\ln(b/a)$ or $\sqrt{ab}\left(1 \pm \frac{\ln b - \ln a}{2}\right)$. As $a = 0.7$ and $b = 0.9$ we get (0.6940; 0.8935)

3.

y_j	2	3	5
$p(y_j)$	0.2	0.6	0.2

$$k_y(x) = \begin{cases} 0 & x < 0 \\ \frac{e^{-2x/y}\,x}{(y/2)^2} = \frac{4x e^{-2x/y}}{y^2} & x > 0 \end{cases} \qquad \text{and then} \qquad k_y(4) = \frac{16e^{-8/y}}{y^2}$$

$$\hat{f}(4) = \sum_{j=1}^{k} p(y_j)k_{y_j}(4) = 0.2\times\frac{16e^{-8/2}}{4} + 0.6\times\frac{16e^{-8/3}}{9} + 0.2\times\frac{16e^{-8/5}}{25} =$$
$$= 0.2\times0.0733 + 0.6\times0.1235 + 0.2\times0.1292 = 0.1146$$

$$K_y(x) = \begin{cases} 0 & x < 0 \\ 1 - \sum_{k=0}^{1}\frac{e^{-2x/y}(2x/y)^k}{k!} = 1 - e^{-2x/y}\left(1 + \frac{2x}{y}\right) & x > 0 \end{cases} \Rightarrow K_y(4) = 1 - e^{-8/y}\left(1 + \frac{8}{y}\right)$$

$$\hat{F}(4) = \sum_{j=1}^{k} p(y_j)K_{y_j}(4) = 0.2\left(1 - e^{-8/2}(1+\frac{8}{2})\right) + 0.6\left(1 - e^{-8/3}(1+\frac{8}{3})\right) + 0.2\left(1 - e^{-8/5}(1+\frac{8}{5})\right)$$
$$= 0.2\times0.9084 + 0.6\times0.7452 + 0.2\times0.4751 = 0.7238$$

$\hat{S}(4) = 1 - \hat{F}(4) = 1 - 0.7238 = 0.2762$ can be computed directly

4.

Empirical percentiles: $n = 11$

$(n+1)\times0.3 = 3.6 \to \tilde{\pi}_{0.3} = 0.4\times4 + 0.6\times8 = 6.4$

$(n+1)\times0.7 = 8.4 \to \tilde{\pi}_{0.7} = 0.6\times23.1 + 0.4\times36.2 = 28.34$

Then, the system to be solved is

$$\begin{cases}\left(\dfrac{6.4}{6.4+\theta}\right)^{\tau}=0.3\\ \left(\dfrac{28.34}{28.34+\theta}\right)^{\tau}=0.7\end{cases}\Leftrightarrow\begin{cases}\tau\left(\ln 6.4-\ln(6.4+\theta)\right)=\ln 0.3\\ \tau\left(\ln 28.34-\ln(28.34+\theta)\right)=\ln 0.7\end{cases}\Leftrightarrow\begin{cases}\tau=\dfrac{\ln 0.3}{\left(\ln 6.4-\ln(6.4+\theta)\right)}\\ \dfrac{\ln 6.4-\ln(6.4+\theta)}{\ln 28.34-\ln(28.34+\theta)}=\dfrac{\ln 0.3}{\ln 0.7}\end{cases}$$

$$\Leftrightarrow\begin{cases}\tau=\dfrac{\ln 0.3}{\left(\ln 6.4-\ln(6.4+\theta)\right)}\\ \left(\ln 6.4-\ln(6.4+\theta)\right)=\left(\dfrac{\ln 0.3}{\ln 0.7}\right)\left(\ln 28.34-\ln(28.34+\theta)\right)\end{cases}$$

$$\Leftrightarrow\begin{cases}\tau=\dfrac{\ln 0.3}{\left(\ln 6.4-\ln(6.4+\theta)\right)}\\ \left(\dfrac{\ln 0.3}{\ln 0.7}\right)\ln(28.34+\theta)-\ln(6.4+\theta)+\ln 6.4-\left(\dfrac{\ln 0.3}{\ln 0.7}\right)\ln 28.34=0\end{cases}$$

To solve the second equation in order to θ we can use EXCEL solver and report the solution to the first equation to obtain an estimate for τ.

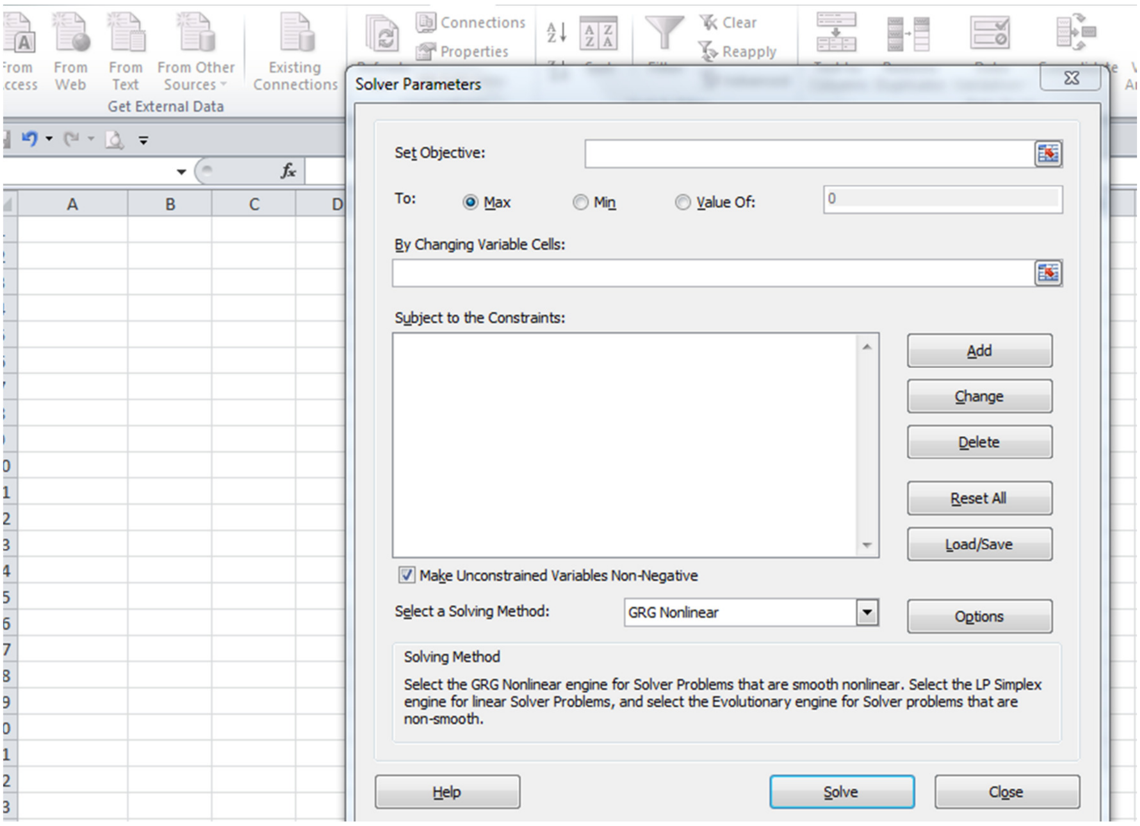
Excel Screen
Cell A1: Theta
Cell B1: 3 an admissible value for Theta
Cell A2: Equation
Cell B2 → =(ln(.3)/ln(.7))*ln(28.34+B1)-ln(6.4+B1)+ln(6.4)- (ln(.3)/ln(.7))*ln(28.34)
Cell A4: Tau
Cell B4: → =(ln(.3)/(ln(6.4)-ln(6.4+B1))

Solver screen:
Set Objective: B2
To: Value of 0

By Changing Variable Cells: B1

Subject to the Constraints: Not needed. Add constraint (B1>=0.000001) if a convergence problem is detected

Solver result: $\hat{\theta}=6.698495$
$\hat{\tau}=1.681058$



5.

a. Maximum likelihood estimate

$$\ell(\theta)=\sum_{i=1}^{5}\ln(\theta^{-2}\,x_i\,e^{-0.5x_i^2/\theta^2})=\sum_{i=1}^{5}\left(-2\ln\theta+\ln x_i-0.5\,x_i^2\,\theta^{-2}\right)$$

$$\ell'(\theta)=\sum_{i=1}^{5}\left(-2\,\theta^{-1}+x_i^2\,\theta^{-3}\right)$$

$\ell'(\theta)=0 \Leftrightarrow 2n\theta^{-1}=\theta^{-3}\sum_{i=1}^{n}x_i^2 \Leftrightarrow \theta^2=\dfrac{\sum_{i=1}^{n}x_i^2}{2n}$ and, as $\theta>0$, we get $\hat{\theta}=\sqrt{\dfrac{\sum_{i=1}^{n}x_i^2}{2n}}$

We can check that $\ell''(\theta)<0$ when $\theta=\hat{\theta}$ as

$\ell''(\theta)=\sum_{i=1}^{n}\left(2\theta^{-2}-3x_i^2\,\theta^{-4}\right)=\theta^{-2}\left(2n-3\theta^{-2}\sum x_i^2\right)$ and then

$$\ell''(\hat{\theta})=\hat{\theta}^{-2}\left(2n-3\frac{2n}{\sum x_i^2}\sum x_i^2\right)=-4n\,\hat{\theta}^{-2}<0$$

The estimate is then $\hat{\theta}=\sqrt{\dfrac{103}{10}}=3.209$

The moment estimator is the solution of $\bar{X}=(\theta/2)\sqrt{2\pi}$, i.e. $\tilde{\theta}=\dfrac{2\bar{X}}{\sqrt{2\pi}}$ and the estimate is $\tilde{\theta}=\dfrac{2\times 4.2}{\sqrt{2\pi}}=3.351$

b.

$$F(x\mid\theta)=\int_0^x\theta^{-2}\,u\,e^{-0.5u^2/\theta^2}\,du=\left(-e^{-0.5u^2/\theta^2}\right]_0^x=1-e^{-0.5x^2/\theta^2}$$

Let β be the 95% VaR of X. Then $F(\beta\,|\,\theta)=0.95$ and consequently

$\beta=\sqrt{-2\theta^2\ln 0.05}=\theta\sqrt{-2\ln 0.05}$ as

$1-e^{-0.5\beta^2/\theta^2}=0.95 \Leftrightarrow e^{-0.5\beta^2/\theta^2}=0.05 \Leftrightarrow -0.5\beta^2/\theta^2=\ln 0.05 \Leftrightarrow \beta^2=-2\theta^2\ln 0.05$ and $\beta>0$

The estimate of the VaR is then $\hat{\beta}=7.856$.

As $\hat{\beta}=\hat{\theta}\sqrt{-2\ln 0.05}$ and $\hat{\text{var}}(\hat{\theta})\approx -1/\ell''(\hat{\theta})=-\dfrac{1}{-4n\hat{\theta}^{-2}}=\dfrac{\hat{\theta}^2}{4n}=\dfrac{10.3}{20}$, we get

$$\hat{\text{var}}\left(\hat{\beta}\right)=\left(\sqrt{-2\ln 0.05}\right)^2 \text{var}(\hat{\theta})=\frac{5.9915\times 10.3}{20}=3.0856$$

Or we can use the delta method defining getting

$$\hat{\text{var}}\left(\beta(\hat{\theta})\right)=\left(\beta'(\hat{\theta})\right)^2 \hat{\text{var}}\left(\hat{\theta}\right)=\left(\sqrt{-2\ln 0.05}\right)^2\frac{\hat{\theta}^2}{4n}=\frac{5.9915\times 10.3}{20}=3.0856$$

c.

$$\begin{aligned}\ell(\theta)&=\ln S(5\,|\,\theta)+\ln f(2\,|\,\theta)-\ln S(1\,|\,\theta)+\ln f(3\,|\,\theta)+\ln f(7\,|\,\theta)-\ln S(2\,|\,\theta)+\ln S(4\,|\,\theta)\\&=-\frac{0.5\times 25}{\theta^2}-2\ln\theta+\ln 2-\frac{0.5\times 4}{\theta^2}+\frac{0.5\times 1}{\theta^2}-2\ln\theta+\ln 3-\frac{0.5\times 9}{\theta^2}-2\ln\theta+\ln 7-\frac{0.5\times 49}{\theta^2}+\frac{0.5\times 4}{\theta^2}-\frac{0.5\times 16}{\theta^2}\\&=-\frac{49}{\theta^2}-6\ln\theta+\ln 2+\ln 3+\ln 7\end{aligned}$$

$$\ell'(\theta)=\frac{98}{\theta^3}-\frac{6}{\theta}$$

$\ell'(\theta)=0 \Leftrightarrow \dfrac{98}{\theta^3}=\dfrac{6}{\theta} \Leftrightarrow \theta^2=\dfrac{98}{6}$ then $\hat{\theta}=\sqrt{\dfrac{98}{6}}\approx 4.041$ as $\ell''(\theta)=-\dfrac{294}{\theta^4}+\dfrac{6}{\theta^2}$ and

$\ell''(\hat{\theta})=-0.7347<0$

6. $f(x\,|\,\theta)=\dfrac{\theta^4}{6}\,x^3\,e^{-\theta x}$, $x>0$, $\theta>0$

a. $L(\theta)=\prod_{i=1}^n f(x_i\,|\,\theta)\propto \theta^{4n}\,e^{-\theta\sum_{i=1}^n x_i}$, $\theta>0$

$\pi(\theta)=\dfrac{1}{25}\theta\,e^{-\theta/5}\propto \theta\,e^{-\theta/5}$, $\theta>0$

$\pi(\theta\,|\,\underline{x})\propto \theta^{4n}\,e^{-\theta\sum_{i=1}^n x_i}\,\theta\,e^{-\theta}=\theta^{4n+1}\,e^{-\theta(0.2+\sum_{i=1}^n x_i)} \qquad \theta>0$

Then $\theta\,|\,\underline{x}\sim \text{G}(4n+2;1/(0.2+\sum_{i=1}^n x_i))$, i.e. $\theta\,|\,\underline{x}\sim \text{G}(202;1/65.2)$

as $n=50$

b. Quadratic loss: $\hat{\theta}_B=E(\theta\,|\,\underline{x})=\dfrac{4n+2}{0.2+\sum x_i}=\dfrac{202}{65.2}=3.098$

0-1 loss function -> mode of the posterior

$$\hat{\theta}_{0-1}=\frac{4n+1}{(0.2+\sum x_i)}=\frac{201}{65.2}=3.082$$

(see Loss Models' appendix)

Using the alternative posterior

$\hat{\theta}_B = E(\theta \mid \underline{x}) = \ 57\times 0.8 = 45.6$ and $\hat{\theta}_{0-1} = 56\times 0.8 = 44.8$

c. As $E(\theta \mid \underline{x}) = \dfrac{4n+2}{0.2+\sum x_i} = \dfrac{202}{65.2} = 3.098$ and

$\mathrm{var}(\theta \mid \underline{x}) = \dfrac{4n+2}{\left(0.2+\sum x_i\right)^2} = \dfrac{202}{65.2^2} = 0.0475$ we get, using Bayes

Central Limit Theorem $3.098 \pm 1.96\sqrt{0.0475}$, i.e., (2.6709; 3.5254)

Using the alternative posterior

$\hat{\theta}_B = E(\theta \mid \underline{x}) = \ 57\times 0.8 = 45.6$ and $\hat{\theta}_{0-1} = 56\times 0.8 = 44.8$

As $E(\theta \mid \underline{x}) =$ 45.6 and $\mathrm{var}(\theta \mid \underline{x}) = 36.48 = 0.0475$ we get (33.76; 57.44)

7. $H_0 : F(x) = \left(\dfrac{x}{x+100}\right)^3$, $x>0$ $\qquad H_1 : H_0$ false

$x_{(i)}$	$(i-1)/5$	$i/5$	$F(x_{(i)})$	D_i
42	0	0.2	0.025875148	0.174125
106	0.2	0.4	0.136243545	0.263756
357	0.4	0.6	0.47671196	0.123288
895	0.6	0.8	0.727779576	0.12778
1334	0.8	1	0.805044773	0.194955

$D = \max D_i = 0.263756$

Approximate critical value: $1.36/\sqrt{5} = 0.6082$

Then we do not reject H_0 ., i.e. we do not reject that the generating distribution is an inverse Pareto with parameters $\tau = 3$ and $\theta = 100$.

8.

a.

1. Determine the number of replicas to be used , NR (a large number)
2. For each replica j
   i. Generate the number of claims, N , according to a Poisson with parameter 0.1. We can generate a pseudo random number and use the inverse transformation method to generate the Poisson (other alternatives like the exponential are possible)
   ii. If we generated $N=0$ in the previous step then define $Y=0$ otherwise generate the individual claim amounts: For each claim we generate a pseudo random number and use the inverse

transformation method to generate the corresponding claim amount. Let u be the pseudo random number. As the claims are Pareto distributed ($\alpha = 3$ and $\theta = 200$) we get

$$u = 1 - \left(\frac{\theta}{\theta + x}\right)^{\alpha} \Leftrightarrow \left(\frac{\theta}{\theta + x}\right)^{\alpha} = 1 - u \Leftrightarrow \frac{\theta}{\theta + x} = (1-u)^{1/\alpha}$$

$$\Leftrightarrow \theta + x = \theta(1-u)^{-1/\alpha} \Leftrightarrow x = \theta(1-u)^{-1/\alpha} - \theta$$

$$\Leftrightarrow x = 200(1-u)^{-1/3} - 200$$

After generating the N claim amounts, we get $Y = \sum_{i=1}^{N} x_i$.

Keep the value of Y as $y_j^{(B)}$

iii. The NR elements of array $y^{(B)}$ are used to approximate the distribution of the aggregate claim amount.

3. To estimate $P(Y > 500)$ count the number of elements of the array $y^{(B)}$ that are greater than 500 and divide by NR.

b.

$u_1 = 0.95$ then $N_1 = 1$ we need to generate one claim amount

$u_2 = 0.85$ then $x = 200(1-0.85)^{-1/2} - 200 = 316.3978$

Then $y_1^B = 316.3978$